Theoretical Model for Drop and Bubble Breakup in Turbulent Dispersions

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A theoretical model for the prediction of drop and bubble (fluid-particle) breakup rates in turbulent dispersions was developed. The model is based on the theories of isotropic turbulence and probability and contains no unknown or adjustable parameters. Unlike previous work, this model predicts the breakage rate for original particles of a given size at a given combination of the daughter particle sizes and thus does not need a predefined daughter particle size distribution. The daughter particle size distribution is a result and can be calculated directly from the model. Predicted breakage fractions using the model for the air–water system in a high-intensity pipeline flow agree very well with the available 1991 experimental results of Hesketh et al. Comparisons of the developed model for specific particle breakage rate with earlier models show it to give breakage-rate values bracketed by other models. The spread in predictions is high, and improved experimental studies are recommended for verification.

Introduction

Turbulent mass transfer in liquid–liquid and gas–liquid dispersed systems is common in the chemical, petroleum, mining, food, and pharmaceutical industries. The possibility of predicting fluid particle (drops or bubbles) size distributions is very important for determining interfacial areas and heat- and mass-transfer rates when designing and scaling up equipment such as chemical reactors and separators (e.g., extractors, distillation columns, and flotation tanks).

Population balances can be used to describe changes in the fluid particle size distributions and other dispersion properties, and are usually the result of dynamic fluid particle breakage and coalescence processes. The main problems in utilizing this technique are to generalize the coalescence and breakup rate models and express them as functions of the basic fluid dynamics and the physical properties of a system. These problems have received considerable attention during the last thirty years. This article focuses only on the rate of drop and bubble breakup in turbulent dispersion systems.

The early work was directed at establishing methods for estimating the maximum stable bubble or drop size, d_m. Shinnar (1961) proposed the following expression for d_m in stirred tanks based on Kolmogorov's concepts:

$$\frac{d_m}{D} \sim We^{-3/5}. \tag{1}$$

Hinze (1955) made a semiquantitative analysis of the forces controlling deformation and breakup of fluid particles and developed methods to estimate a stable bubble or drop size in a dispersion system relying on two dimensionless groups: a Weber group and a viscosity group. Based on this concept, Hughmark (1971) suggested the following correlation for d_m in turbulent pipe flows:

$$1.69 d_m = \frac{\sigma}{\rho_c \hat{u}^2} + \frac{1.3 \mu_d}{(\rho_c \rho_d)^{1/2} \hat{u}}. \tag{2}$$

Previous work on breakup rates

Considerable effort has been spent in detailed analysis and modeling of the breakup rate process. Valentas et al. (1966) proposed a purely empirical correlation for the specific drop breakup rate:

$$\frac{\Omega_B(d)}{n} = c_1 d^{c_2}, \qquad c_2 = 0, \frac{1}{3}, \frac{2}{3}, 1.$$

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Ross and Curl (1973) used an analog to the "activated complex" concept from chemical-reaction kinetics and obtained the relationship:

$$\frac{\Omega_B(d)}{n} = c_1\left(\frac{\epsilon}{d^2}\right)^{1/3} \exp\left(-\frac{c_2\sigma}{\rho_c \epsilon^{2/3} d^{5/3}}\right). \tag{4}$$

Coulaloglou and Tavlarides (1977) assumed identical kinetic energy distributions for drops and turbulent eddies in order to develop drop breakup efficiencies. They also assumed the motion of daughter drops to be similar to that of turbulent eddies and could thereby estimate a "characteristic breakup time." Based on this, a drop breakup model, nearly identical to that of Ross and Curl (1973), was obtained for stirred tanks.

Narsimhan et al. (1979) gave a more theoretical analysis of the processes leading to drop breakage. Based on probability theory and such assumptions as the number of eddies arriving at the surface of a droplet being a Poisson process, the arrival frequency of eddies being constant, they proposed a binary drop-breakage model as

$$\frac{\Omega_B(d)}{n} = \Lambda \,\mathrm{erfc}\left(\frac{A\sigma^{1/2}}{\rho_c^{1/2}\epsilon^{1/3}d^{5/6}}\right), \tag{5}$$

where $A = [3(2^{2/3}-1)]^{1/2}$ and Λ is the average frequency of eddies arriving at a drop surface. Narsimhan et al. (1979) assumed that Λ was independent of drop size.

In 1983 Chatzi (Chatzi and Lee, 1987) assumed a breakage mechanism in which a drop will break if its turbulent kinetic energy is greater than its surface energy, and gave a drop breakage rate model in stirred tanks as

$$\frac{\Omega_B(d)}{n} = c_1\left(\frac{\epsilon}{d^2}\right)^{1/3} \Gamma\left(\frac{3}{2}, \frac{c_2\sigma}{\rho_d \epsilon^{2/3} d^{5/3}}\right). \tag{6}$$

Lee et al. (1987a) also developed a bubble breakage model based on the work of Narsimhan et al. (1979) using dimensional analysis to obtain an expression for the average frequency of eddies, Λ, arriving at a drop surface.

$$\frac{\Omega_B(d)}{n} = c_1\left(\frac{\epsilon}{d^2}\right)^{1/3} \left[1 - \int_0^1 F\left(\frac{c_2\sigma}{\rho_d \epsilon^{2/3} d^{5/3} \phi^{11/3}}\right) d\phi\right], \tag{7}$$

where $F(\ )$ is the cumulative chi-square distribution function.

Hesketh et al. (1991b) combined the natural oscillation mode of a sphere given by Lamb (1932) and a correlation for the maximum stable drop size in a stirred tank developed by themselves, and obtained an empirical drop breakage rate:

$$\frac{\Omega(d)}{n} \approx 2.7\left(\frac{\rho_c^{0.1}\rho_d^{0.3}\epsilon^{0.6}}{\sigma^{0.4}}\right). \tag{8}$$

Previous work on daughter particle-size distributions

As pointed out by Valentas et al. (1966), a complete description of drop or bubble breakup processes also needs the so-called "daughter drop- or bubble-size distribution," because many combinations of daughter particle sizes can occur after breakage, ranging from two particles of equal size to one very small and one very large particle. All the previously cited rate models only give the total breakup rate for a given parent particle size and say nothing about the resulting daughter particle sizes. Therefore, most of the previous models rely on direct assumptions regarding the daughter size distributions. In addition, they are empirical in nature, and usually two or more unknown parameters need to be determined.

For binary breakage, Valentas et al. (1966) assumed a delta function as the discrete breakup daughter particle-size distribution, and a truncated normal density function for the continuous breakup daughter size distributions. The truncated normal function has also been used by other authors, such as Coulaloglou and Tavlarides (1977), Chatzi et al. (1989), and Chatzi and Kiparissides (1992).

Narsimhan et al. (1979) and Randolph (1969) assumed that a uniform distribution could be used, while Lee et al. (1987b) used a beta distribution function. All the functions mentioned earlier, except the uniform distribution, have the same characteristics: a decreasing breakage percentage appears when $v_l \to 0$ or v, while the equal-sized breakage has the highest probability.

As pointed out by Nambiar et al. (1992), the models that assume a uniform or a truncated normal function-like distribution, centered at $v/2$, for the daughter bubble or drop size, may not be representative of the underlying physical situation. The physical concept is clear: more energy is required for binary equal-sized breakage than binary unequal-sized breakage. This is also supported by the experimental results of Hesketh et al. (1991a) for bubble and drop breakage in turbulent pipe flows. These results show that equal-sized breakage has the lowest breakage probability while the highest breakage likelihood occurs when $v_l \to 0$ (or $\to v$).

Hence, for the daughter particle-size distribution, Hesketh et al. (1991b) proposed a so-called $1/X$-shaped function with an adjustable parameter determined by a best fit to their experimental data. However, the $1/X$-shaped function has a zero probability for equal-sized breakage, which is in contradiction with their own experimental results.

Recently, Nambiar (1992) proposed a method to predict the daughter drop-size distribution for drop breakage in turbulent stirred dispersions based on an eddy interaction model, but this method still predicts a zero probability for equal-sized breakage. In their coalescence model, the interaction time correlation proposed by Levich (1962), based on dimensionless analysis, was also needed.

Purpose of this work

As seen from the preceding review, previous breakup models all include two or more unknown parameters that need to be determined by experimental work. This may involve costly experimental programs, and may also reduce the generality of the models.

Thus, this article is aimed at developing a more fundamental rate model for drop or bubble breakage in turbulent fluid–fluid dispersion systems.

Breakup Rate

Phenomenological simplifications

In turbulent dispersion systems, the fluid dynamics and breakup processes are complex. Hence, in order to develop the breakage model, certain simplifications are necessary.

1. To make the problem tractable, the turbulence is usually assumed to be isotropic. This assumption is fairly acceptable for stirred-tank systems. For other systems, like bubble columns, the turbulence is nonisotropic (Menzel, 1990). Nevertheless, the isotropic turbulence assumption has often been used also for these systems (e.g., Lee et al., 1987a; Prince and Blanch, 1990). This is because theoretical considerations and experimental evidence have shown, as concluded by Hinze (1959), that the fine-scale structure of most actual nonisotropic turbulent flows is locally nearly isotropic. Many features of isotropic turbulence may thus be applied to phenomena in actual turbulence that are determined mainly by the fine-scale structure (Hinze, 1959). Furthermore, even an actual turbulence situation with a nonisotropic large-scale structure, or that is nonisotropic through an essential part of its spectrum, can often, as a first approximation, be treated as if it were isotropic. The differences between results based upon the assumed isotropy and actual results are often sufficiently small to be disregarded compared to the uncertainty of the experimental data (Hinze, 1959).

2. Only the binary breakage of fluid particles in a turbulent dispersion is considered. As is well known, a bubble or drop may break into two or more particles with equal or unequal volumes, depending on the breakage type. It is commonly believed that more than one mechanism for particle breakage may exist in turbulent dispersions, since a drop or bubble is not only exposed to a turbulent field, but is also subjected to both inertial and viscous forces. Normally, two are considered to be important, these being turbulent (deformation) breakage and viscous shear (tearing) breakage (Sleicher, 1962; Collins and Knudson, 1970; Walter and Blanch, 1986; Hesketh et al., 1991a).

The turbulent breakage is induced by fluctuating eddies bombarding the particle surface (Walter and Blanch, 1986) causing oscillations (or deformations) of the particle surface. That is, a fluid particle of sufficient size will oscillate around its equilibrium shape. The oscillations are brought about by the kinetic energy of the turbulent motion in the continuous phase, or by the relative velocity fluctuations between points in the close vicinity of the particle surface. In other words, the kinetic energy of the turbulent motion brings about an increase in the surface energy of the particle through deformations. Fragmentation of the particle occurs if the turbulent motion provides an increase in surface energy sufficient to cause breakage. Usually, binary breakage occurs in this case.

For the shear breakage, a drop or bubble may break into several drops or bubbles with varying volumes due to viscous shear. However, when concerned with bubble or drop breakage in highly turbulent flow, the viscous forces can usually be neglected, as the bubbles and drops are usually much larger than the microscale of turbulence (Shinnar, 1961; Narsimhan et al., 1979).

The simplification of binary breakage has been used by many authors (e.g., Narsimhan et al., 1979; Hesketh, 1991a; Nambiar et al., 1992). The recent experimental results of Hesketh et al. (1991a) support the assumption. They found that all bubble and drop breakage events were binary in turbulent pipeline flows. In the present model, the assumption of binary breakage is also used. In this context this means that multiple breakages on the same particle do not occur simultaneously. This is partly done to simplify the model, but also from the viewpoint that in the process of bubble or drop breakage into more than two bubbles/droplets, the likelihood of two or more breakage events taking place precisely at the same time is very small.

3. The breakage volume fraction is assumed to be a stochastic variable. For binary breakage, a dimensionless variable describing the sizes of daughter drops or bubbles (the breakage volume fraction) can be defined as

$$f_{BV} = \frac{v_1}{v} = \frac{d_I^3}{d^3} = \frac{d_I^3}{d_I^3 + d_{II}^3}, \tag{9}$$

where d_I and d_{II} are diameters (corresponding to volumes v_I and v_{II}) of the daughter particles in the binary breakage of a parent particle with diameter d (corresponding to volume v). Obviously, the value interval for the breakage volume fraction should be $0 < f_{BV} < 1$ ($f_{BV} = 0.5$ meaning equal binary breakage and $f_{BV} = 0$ or 1 meaning no breakage).

Fluid particles will break into a preferred range of daughter particle sizes depending on the flow conditions. However, Hesketh et al. (1991a) found that there was no correlation between the range of f_{BV} values and the size of the parent particle. In addition, no evidence exists that there is a relationship between the breakage volume fraction and the eddy sizes, although Nambiar et al. (1992) attempted to develop such a relationship (e.g., an eddy of size equal to the drop diameter was considered to split the drop into two equal fragments).

Thus, as an approximation, in this work it is assumed that the breakage volume fraction, f_{BV}, is a stochastic variable.

4. The occurrence of breakup is determined by the energy level of the arriving eddy. Analogous to droplets in turbulence, according to Narsimhan et al. (1979), oscillations of a bubble or drop induced by a particular eddy, may be changed by the arrival of other eddies. The degree of interference is largely determined by the frequency of the eddy arrival process compared to the already existing oscillations of the particle. If the two frequencies are comparable, the effects of successive eddies continually interfere with each other. Since particle oscillations, sufficiently vigorous to make the particle break, are caused only by hits of eddies with scale similar to, or smaller than the particle diameter, then the smaller particles require very small eddies to induce oscillations. Furthermore, the time scale of oscillations should be inversely proportional to the eddy frequency. Thus the smaller eddies will be expected to create high-frequency oscillations.

In this work, the assumption also used by Narsimhan et al. (1979) and Lee et al. (1987a), is employed: the particle oscillation frequency is larger than the arrival frequency of eddies. This implies that the eddies affect the particles independently such that once an eddy of sufficiently high energy arrives, the particle will break.

5. Only eddies of length scale smaller than or equal to the particle diameter can induce particle oscillations. Nambiar et al. (1992) discuss the movement of particles in a field of homogeneous turbulence. When the Reynolds number is large enough, the large eddies are responsible for most of the translatory motion of particles, while small eddies determine the strain experienced by the individual particle or group of particles. The latter thus dominate the deformation of the particles in the flow field. It may therefore be reasonable to assume that only eddies of length scale smaller than or equal to the particle diameter can participate in its deformation and that the larger eddies merely convect the particle.

Basic model of breakage rate

In a turbulent field, velocity fluctuations at a point can be thought of as caused by the arrival of eddies of a spectrum of lengthscales (frequencies). Similarly, the fluctuations in relative velocity on the surface of a bubble or drop exposed to a turbulent field can be considered to be due to the arrival of similar eddies at the surface. This is equivalent to the "bombardment" of eddies on a particle surface. Of course, the various arriving eddies contain different amounts of energy, and thus will provide or supply different energies to the surface energy required for fragmentation of a particle.

The kinetic energy contained in an eddy and the number density of the eddy depend on its length scale. Since the required energy for a particle breakage depends on the breakage volume fraction, f_{BV}, we propose a general rate model for fluid particle breakage in turbulence as follows:

$$\Omega_B(v:vf_{BV}) = \int_{\lambda_{\min}}^{d} P_B(v:vf_{BV},\lambda)\,\dot{\omega}_{B,\lambda}(v)\,d\lambda \qquad (10)$$

Here $\dot{\omega}_{B,\lambda}(v)$ is the arrival (bombarding) frequency of eddies of size (length scale) between λ and $\lambda + d\lambda$ onto particles of size v; $P_B(v:v_I,\lambda)$ is the probability for a particle of size v to break into two particles, one with size (volume) $v_I = vf_{BV}$, when the particle is hit by an arriving eddy of size λ. According to assumption 4 in the preceding subsection, $P_B(v:v_I,\lambda)$ will equal the probability of the arriving eddy of size λ having a kinetic energy greater than, or equal to, the minimum energy required for particle breakage to occur. The upper limit for the integral in Eq. 10 is based on simplification (assumption 5) in the same subsection.

One should note that since f_{BV} is considered to be the independent variable with range (0,1), then $\Omega_B(v:vf_{BV})$ is the breakage rate of particles of size v into a fraction between f_{BV} and $f_{BV} + df_{BV}$ (one of the daughter particles has a volume between vf_{BV} and $vf_{BV} + vdf_{BV}$) for a continuous f_{BV}, and into a fraction equal f_{BV} for a discrete f_{BV}, respectively.

Arrival or bombarding frequency of eddies

The arrival frequency of eddies with a given size λ on the surface of drops or bubbles with size d, is equivalent to the collision frequency between the same eddies and fluid particles. Since the motion of eddies is considered random, the collision frequency of eddies of a size between λ and $\lambda + d\lambda$ with particles of size d can be expressed by

$$\dot{\omega}_{B,\lambda}(d) = \frac{\pi}{4}(d+\lambda)^2 \bar{u}_\lambda \dot{n}_\lambda n, \qquad (11)$$

where $\dot{n}_\lambda$ is the number of eddies of size between λ and $\lambda + d\lambda$ per unit reactor volume, and $\bar{u}_\lambda$ is the turbulent velocity of eddies of size λ. This is also considered to be the relative velocity between particle and eddy.

The mean turbulent velocity of eddies with size λ in the inertial subrange of isotropic turbulence can be expressed by (Kuboi et al. (1972a,b)

$$\bar{u}_\lambda = \left(\frac{8\overline{u^2}}{3\pi}\right)^{1/2} = \left(\frac{8\tilde{\beta}}{3\pi}\right)^{1/2}(\epsilon\lambda)^{1/3} = \beta^{1/2}(\epsilon\lambda)^{1/3}, \qquad (12)$$

where the constant, $\tilde{\beta} = (3/5)\Gamma(1/3)\alpha$. Here α is considered to be a universal constant, as given by Batchelor (1982), based on turbulence theory. $\tilde{\beta}$ becomes about 2.41 when using $\alpha = 1.5$ (Tennekes and Lumley, 1972). The measured value of $\tilde{\beta}$ is 2.0 according to Kuboi et al. (1972a).

The energy spectrum, $E(k)$, gives the kinetic energy contained in eddies of wave number between k and $k + dk$, or equivalently, of size between λ and $\lambda + d\lambda$, per unit mass (Tennekes and Lumley, 1972). When this is known, a relationship between $\dot{n}_\lambda$ and $E(k)$ can be obtained as follows

$$\dot{n}_\lambda \rho_c \frac{\pi}{6}\lambda^3 \frac{\bar{u}_\lambda^2}{2} d\lambda = E(k)\rho_c(1-\epsilon_d)(-dk), \qquad (13)$$

where ϵ_d is the local fraction of dispersed phase.

The functional form of the energy spectrum for the whole range of isotropic turbulence is not available, but in the inertial subrange it is well described (Tennekes and Lumley, 1972) by

$$E(k) = \alpha\epsilon^{2/3}k^{-5/3}. \qquad (14)$$

The relationship between the wave number and the size of an eddy is $k = 2\pi/\lambda$ (Tennekes and Lumley, 1972). Therefore, the number of eddies of size between λ and $\lambda + d\lambda$ per unit reactor volume, or the number density of eddies, is

$$\dot{n}_\lambda = \frac{c_3(1-\epsilon_d)}{\lambda^4}, \qquad (15)$$

where

$$c_3 = \frac{9\alpha}{2(2\pi)^{2/3}\tilde{\beta}} = \frac{15}{2(2\pi)^{2/3}\Gamma(1/3)} \approx 0.822. \qquad (16)$$

Equation (15) indicates that smaller eddies have higher number densities. However, the equation is only valid for eddies in the inertial subrange of isotropic turbulence because the used turbulent energy spectrum function and the turbulent velocity are only valid in this subrange. This limitation will probably have an insignificant effect on the eddy bombarding consideration, since the very small eddies have very low energy contents and very short lifetimes.

Consequently, the bombarding frequency of the eddies with size between λ and $\lambda + d\lambda$ on particles of size d can be expressed as

$$\dot{\omega}_{B,\lambda}(d) = \dot{\omega}_{B,\xi}(\xi) = c_4(1-\epsilon_d)n(\epsilon d)^{1/3}\frac{(1+\xi)^2}{d^2\xi^{11/3}}, \quad (17)$$

where $\xi = \lambda/d$ is the size ratio between an eddy and a particle, and

$$c_4 = c_3\pi\beta^{1/2}/4 \approx 0.923. \quad (18)$$

Breakage probability (efficiency)

For a particular eddy hitting a particle, the probability for particle breakage depends not only on the energy contained in the arriving eddy, but also on the minimum energy required by the surface area increase due to particle fragmentation. The latter is determined by the number and the sizes of the daughter particles formed in the breakage processes.

To determine the energy contained in eddies of different scales, a distribution function of the kinetic energy for eddies in turbulence is required. Lee et al. (1987a) used Maxwell's law for this function. However Maxwell's law is especially for free-gas molecular motion and may not be suitable for turbulent eddies. Angelidou et al. (1979) have developed an energy-distribution density function for fluid particles in liquids, which satisfies a natural exponential function. Actually, for the kinetic energy of turbulent eddies, this exponential-energy density function is found to be equivalent to the common assumption that the velocity distribution of turbulent eddies is a normal density function (Saffman and Turner, 1956; Coulaloglou and Tavlarides, 1977; Narsimhan et al., 1979). This assumption of a normal velocity distribution is also supported by the experimental results of Kuboi et al. (1972a) for a turbulent liquid–liquid dispersion system. Hence, this distribution function is also used in the present work to describe the kinetic energy distribution of the eddies in turbulence:

$$p_e(\chi) = \frac{1}{\bar{e}(\lambda)}\exp(-\chi), \qquad \chi = \frac{e(\lambda)}{\bar{e}(\lambda)}. \quad (19)$$

Here the mean kinetic energy of an eddy with size λ, $\bar{e}(\lambda)$, is given by

$$\bar{e}(\lambda) = \rho_c\frac{\pi}{6}\lambda^3\frac{\bar{u}_\lambda^2}{2} = \frac{\pi\beta}{12}\rho_c(\epsilon d)^{2/3}d^3\xi^{11/3}. \quad (20)$$

When a particle of size d breaks into two particles with a given value of f_{BV}, the increase in surface energy is

$$\bar{e}_i(d) = \left[f_{BV}^{2/3} + (1-f_{BV})^{2/3} - 1\right]\pi d^2\sigma = c_f\pi d^2\sigma, \quad (21)$$

where c_f is defined as the increase coefficient of surface area, that is

$$c_f = f_{BV}^{2/3} + (1-f_{BV})^{2/3} - 1. \quad (22)$$

As seen, $c_f(0 \le c_f \le 2\cdot 0.5^{2/3} - 1)$ depends only on the breakage volume fraction, f_{BV}, and is a function that is symmetrical about $f_{BV} = 0.5$.

Since the time scale of particle oscillations is assumed to be smaller than that associated with the eddy bombardment, implying that once an eddy of sufficiently high energy arrives, this leads to particle breakage, then the condition for an oscillating deformed particle to break is that the kinetic energy of the bombarding eddy exceeds the increase in surface energy required for breakage:

$$e(\lambda) \ge \bar{e}_i(d) = c_f\pi d^2\sigma. \quad (23)$$

Consequently, according to probability theory, the probability for a particle of size v or d to break into a size of $v_I = vf_{BV}$ when the particle is hit by an arriving eddy of size λ, will be equal to the probability of the arriving eddy of size λ having a kinetic energy greater than or equal to the minimum energy required for the particle breakup. This gives

$$P_B(v{:}vf_{BV},\lambda) = P_e[e(\lambda) \ge \bar{e}_i(d)] = P_e[\chi \ge \chi_c] = 1 - P_e[\chi \le \chi_c], \quad (24)$$

where χ is the dimensionless energy, $e(\lambda)/\bar{e}(\lambda)$, and χ_c is the critical dimensionless energy for breakup:

$$\chi_c = \frac{\bar{e}_i(d)}{\bar{e}(\lambda)} = \frac{12c_f\sigma}{\beta\rho_c\epsilon^{2/3}d^{5/3}\xi^{11/3}}. \quad (25)$$

Then, the conditional breakage probability, $P_B(v{:}vf_{BV},\lambda)$, can be expressed as

$$P_B(v{:}vf_{BV},\lambda) = 1 - \int_0^{\chi_c}\exp(-\chi)\,d\chi = \exp(-\chi_c). \quad (26)$$

The expression for breakage rate

Substituting Eqs. 17 and 26 into Eq. 10, the breakup rate of particles of size v or d into particle sizes of vf_{BV} and $v(1-f_{BV})$ can be obtained as

$$\frac{\Omega_B(v{:}vf_{BV})}{(1-\epsilon_d)n} = c_4\left(\frac{\epsilon}{d^2}\right)^{1/3}\int_{\xi_{\min}}^{1}\frac{(1+\xi)^2}{\xi^{11/3}}\exp\left(-\frac{12c_f\sigma}{\beta\rho_c\epsilon^{2/3}d^{5/3}\xi^{11/3}}\right)d\xi, \quad (27)$$

where $\xi_{\min} = \lambda_{\min}/d$.

In the preceding integral the microscale of eddies, λ_d, should actually be used as the lower limit, but it has been replaced by the minimum size of eddies in the inertial subrange of isotropic turbulence, $\lambda_{\min}$. The reason is that the expressions for bombarding frequency of eddies and breakage probability developed earlier are only valid for this subrange. However, as discussed previously, this change is acceptable since the very small eddies have very low energy contents and very short lifetimes, thereby having a negligible effect on the breakage of particles.

Tennekes and Lumley (1972) have given the minimum size of eddies in the inertia subrange as $2\pi\lambda_d/\lambda_{\min} \approx 0.2$–$0.55$ or $\lambda_{\min}/\lambda_d \approx 11.4$–$31.4$.

The integrand in Eq. 27 can be expressed by the incomplete gamma functions and is then easy to calculate.

As mentioned before, since f_{BV} is the independent variable in the interval (0,1), then $\Omega_B(v:vf_{BV})$ represents the breakage rate for particles of size v into fractions represented by an f_{BV} between f_{BV} and $f_{BV}+df_{BV}$ for a continuous f_{BV} function. Thus, the total breakage rate of particles of size v or d can be obtained by integrating the preceding equation over the whole interval. The total breakage rate of particles of size v or d is then expressed as

$$\Omega_B(v)=\frac{1}{2}\int_0^1\Omega_B(v:vf_{BV})\,df_{BV}, \tag{28}$$

where the factor 1/2 takes into account that the effective range of f_{BV} is either 0–0.5 or 0.5–1 (the integrand is symmetrical with $f_{BV}=0.5$).

Daughter Particle Size Distribution

As mentioned before, all previous models for breakage rate depend on a predefined daughter particle size distribution. This is because the rate models can only give the total breakage rate for particles of size v, $\Omega_B(v)$. The daughter particle size distribution, $\eta(v:v_I)$, was first introduced by Valentas et al. (1966) to describe the size distribution of daughter drops or bubbles. It was also called the "breakage kernel." For a continuous daughter particle size distribution, $\eta(v:v_I)dv_I$ represents the fraction of particles of size v that break into particles of size between v_I and v_I+dv_I.

However, as mentioned before, the choice of daughter particle size distribution has usually been more or less arbitrary by previous authors. Most of the functions used, except the uniform distribution, have the same characteristics; a decreasing breakage percentage appears when $v_I\to 0$ or v, while the equal-sized breakage has the highest probability. Nambiar et al. (1992) have pointed out that the models that have hitherto assumed a uniform or a truncated normal function-like distribution, centered at $v/2$ for the daughter bubble or drop size, may not be representative of the underlying physical situation. This is correct in a physical sense because more energy is required for binary equal-sized breakage then binary unequal-sized breakage. The experimental results of Hesketh et al. (1991a) have also shown that equal-sized breakage has the lowest breakage probability, while the highest breakage likelihood occurs when $v_I\to 0$ (or $\to v$).

Unlike previous work, the present model does not need the daughter particle size distribution because the model directly gives the "partial breakage rate" for particles of size v breaking into the daughter particles with a given f_{BV}. The daughter particle size distribution now comes out as a result and can be calculated directly from this model.

For a continuous f_{BV}, the $\Omega_B(v:vf_{BV})$ describes the rate at which particles of size v or d break into a size between v and v_I+dv_I ($v_I=vf_{BV}$). Then, according to the definition, the daughter particle size distribution is given by

$$\eta(v:vf_{BV})=\frac{2\int_{\xi_{\min}}^1(1+\xi)^2\xi^{-11/3}e^{-\chi_c}\,d\xi}{v\int_0^1\int_{\xi_{\min}}^1(1+\xi)^2\xi^{-11/3}e^{-\chi_c}d\xi\,df_{BV}}. \tag{29}$$

Results and Discussion

In contrast to previous models for bubble or drop breakage, the present model for fluid particle breakup has no unknown or tuned parameters. Both the breakage rate and the daughter bubble or drop size distribution can be predicted, given the operating conditions and the fluid system.

The present model shows that the particle sizes resulting from a bubble or drop breakage are normally functions of the original bubble size, the energy dissipation rate, and the physical properties. The dimensionless daughter bubble size distribution ηv, for the air–water system, is illustrated in Figure 1. It shows that the dimensionless daughter size distribution is a U-shaped function and that the lowest probability (which is nonzero) is found for equal-sized breakage for any given original particle size. This agrees well with both physical intuition and the experimental results obtained by Hesketh (1991a,b).

The daughter bubble size distribution depends, as mentioned earlier, not only on the energy dissipation rate, but also on the original bubble size. As seen from Figure 1, the curves for 6-mm bubbles are flatter than for 3-mm bubbles, and the difference between the two energy dissipation rates is smaller for the larger bubble size. For even larger bubbles, the effect of the energy dissipation rate becomes insignificant and the daughter bubble size distribution tends to become flat. This situation can be seen more clearly in Figure 2, where the breakage fraction is given as a function of breakage volume fraction, f_{BV}. The breakage fraction is defined as the number fraction of original bubbles, breaking into a certain distribution of daughter bubbles defined by f_{BV}. The fractions are summed over every interval of 0.05 in f_{BV}. For 3-mm bubbles and a low-energy dissipation rate (0.5 m^2/s^3), the breakage fraction is high, (~ 0.6) for unequal-sized breakage, and goes down to about 0.02 for equal-sized breakage, whereas for 6-mm bubbles at $\epsilon=1$ m^2/s^3, the same span is from 0.29 to 0.07. This is physically reasonable, since there is a wider size range of eddies affecting the larger bubbles, and therefore a higher chance of breaking them into daughter bubbles with close to equal sizes. An increase in the energy

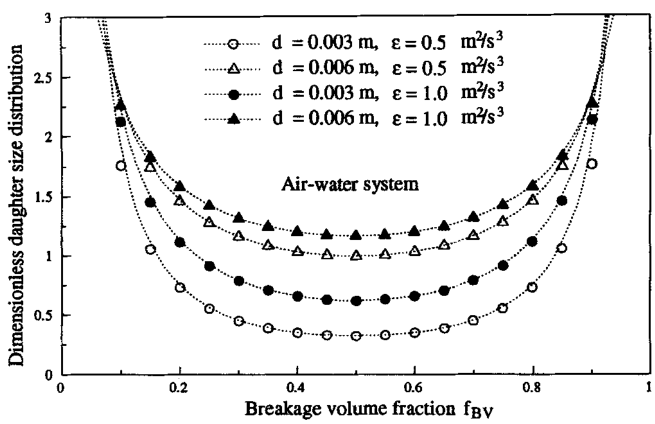

Figure 1. Effect of bubble size and energy dissipation rate per unit mass, on the dimensionless daughter bubble-size distribution, $\eta(v:vf_{BV})v$, for the air–water system.

Symbols do not represent experimental points, but distinguish curves.

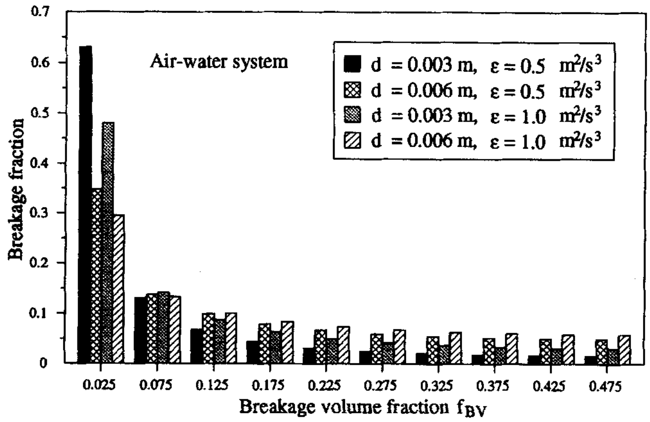

Figure 2. Effect of bubble size and energy dissipation rate per unit mass, on the breakage fraction as function of the breakage volume fraction for the air–water system.

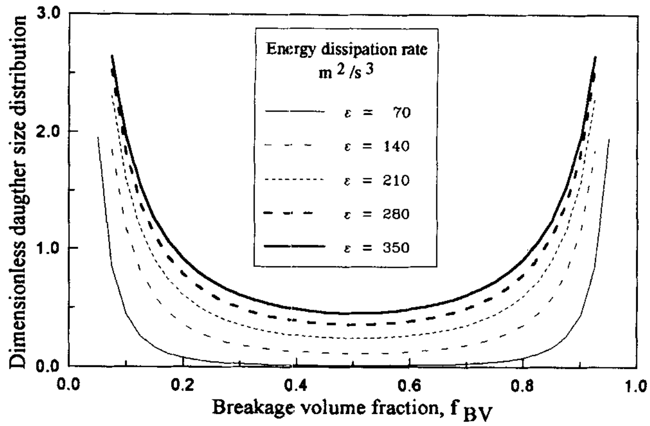

Figure 3. Effect of bubble size and energy dissipation rate per unit mass, on the dimensionless daughter bubble-size distribution, $\eta(v{:}vf_{BV})v$, for the system oil–water.

Interfacial tension oil–water $\sigma_{12} = 0.05$ N/m.

dissipation rate also makes the distribution flatter since this is equivalent to providing a higher energy for breakage. It is also consistent with the distinction made by Narsimhan et al. (1984) between "thorough" and "erosive" breakages, where "erosive" breakages, meaning separations of small particles, was found to be more common. For large particles, however, the chances of equal-sized, "thorough" breakage became more probable.

From Figures 1 and 2 it is seen that the prevalent breakage volume fraction lies in the low range of daughter bubble sizes. This means that most frequently small bubbles separate from the original larger bubble. The separation of a small bubble from a large bubble may of course take place many times in rapid succession, and thus resemble the breakage into more than two bubbles. There is no inherent assumption made in the present model preventing a fluid particle from being hit by more than one eddy almost at the same time. In fact, multiple breakage may be caused by a close to "simultaneous" bombardment of several smaller eddies. In this way, the model may be thought of as also covering the erosion type of breakage that is described by Chatzi and Kiparissides (1992). However, basically, each separation is modeled as a binary breakup.

In liquid–liquid dispersions in stirred tanks, smaller droplet sizes prevail. Model results for the case of small oil droplets ($d = 200$ μm) in water are shown in Figure 3. As expected the energy dissipation rates needed to break these smaller droplets are much greater than for the air–water system. Energy dissipations rates in the range 70–350 m^2/s^3 were used. This is the range that would be expected in the region near the impeller in a stirred tank. According to Keey (1967), dissipation rates in this region may be 70 times higher than the average value. It is clearly seen that the probability for unequal breakage increases dramatically, compared to the air–water system. At smaller droplet diameters, the effect is even greater. Also for these droplet sizes the model predicts a flattening of the daughter droplet size distribution with increasing dissipation rate.

In order to test the present breakage rate model against experimental results, the predicted breakage fractions for the air–water system in pipeline flows have been compared with the measured results of Hesketh et al. (1991a,b), as shown in Figure 4, which also illustrates the predicted results. It can be seen that the agreement between the predicted results of the present model and the experimental results is very good. In addition, it can be found that the original bubble size, *d*, has nearly no effect on the daughter bubble distribution or breakage kernel, under the high-energy dissipation rates, $\epsilon = 13.3$ m^2/s^3, which prevailed during the experiments. Furthermore, unlike the models of Hesketh et al. (1991b) and Nambiar et al. (1992), the present model shows that the fraction of equal-sized breakage is nonzero, except for very small bubbles (close to the minimum length scale of eddies in a system) or at very low-energy dissipation rates. This is also supported by the experimental results of Hesketh et al. (1991a).

The influence of bubble size and energy dissipation rate on the specific breakage rate, $\Omega_B/[(1-\epsilon_d)n]$, is shown in Figure 5, with the air–water system as an example. The larger the bubble size and/or the energy dissipation rate, the higher the

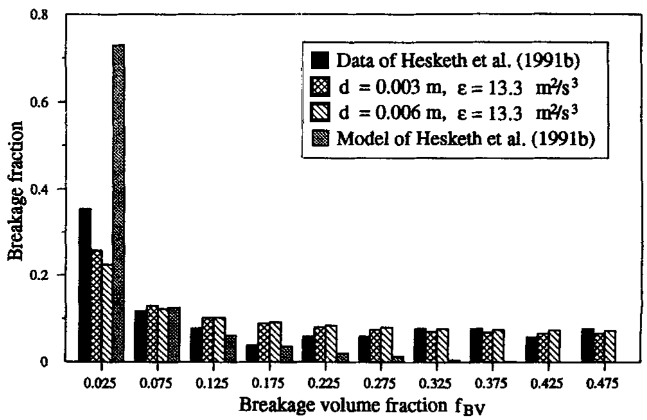

Figure 4. Comparison of predicted breakage fractions for two bubble sizes from the developed model with the measured data of Hesketh et al. (1991a).

The system is air–water.

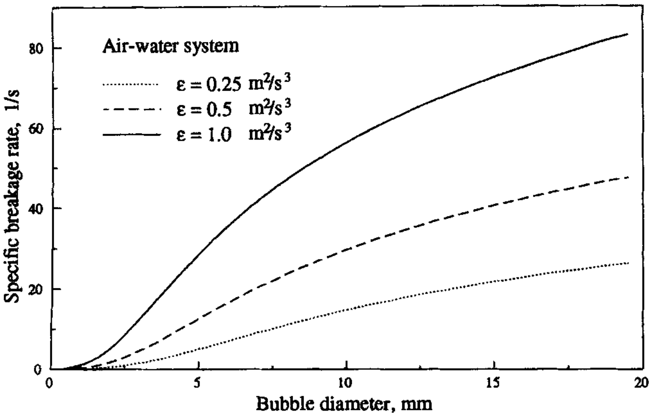

Figure 5. Effect of bubble size and energy dissipation rate per unit mass on the specific breakage rate, $\Omega_B/[(1-\epsilon_d)n]$, for the air-water system.

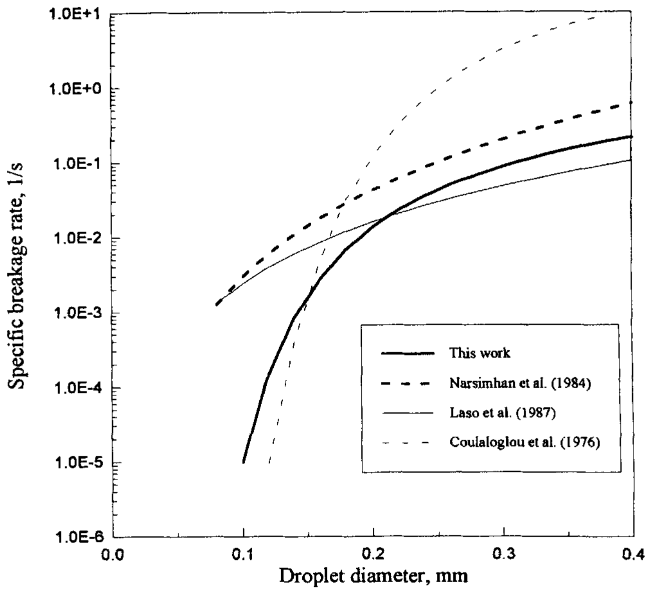

Figure 7. Comparison between various models for the specific breakage rate, $\Omega_B/[(1-\epsilon_d)n]$, as function of droplet diameter.

The system is oil-water with interfacial tension $\sigma_{12} = 0.05$ N/m.

specific breakage rate. This is reasonable, as pointed out before, since a larger bubble can be hit by a wider range of eddies, and a larger energy dissipation rate means a higher energy content per unit mass of eddies. The specific breakage rate of very small bubbles is close to zero, because the eddies capable of causing the bubbles to oscillate are too small to make them break. As the energy dissipation rate increases, the bubble size under which no breakage occurs, is decreased. The same tendency is shown in Figure 6 for small (200 μm) oil droplets in water, but even more pronounced. In this case, the breakage rate is generally much lower than for bubbles in water because of the increased density and viscosity of the fluid particles. Also the energy dissipation level needed for breakup in this size range is higher since higher energy levels are needed to break smaller drops.

Figure 7 contains a comparison between the present model for specific breakage particle rate and models found in the literature. The literature models, Narsimhan et al. (1984), Laso et al. (1987), Coulaloglou and Tavlarides (1976, 1977), Chatzi et al. (1989), and Chatzi and Kiparissides (1992), are all fitted to data from stirred-tank experiments using population balance modeling, and they contain one or more parameters. In addition, the geometric dimensions of the tank/impeller system enter into the formula in different ways. The numerical comparison in Figure 7 is thus done at a fixed average energy dissipation rate ($\epsilon = 1$ m^2/s^3), but with the pertinent geometric data given for each reference. The figure shows that the spread in predicted breakage rates is wide. This is similar to what was found by Laso et al. (1987), and they attributed the inconsistencies to differences in the experimental conditions. This may be true, as all models are extensive in nature and thus tailored to the equipment used. However, the underlying model assumptions also vary, in particular the treatment of the coalescence processes taking place.

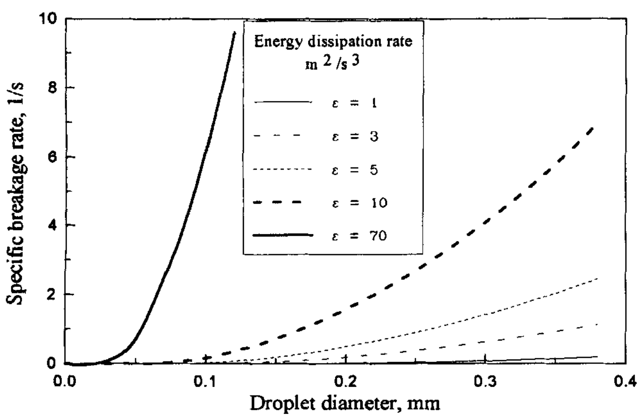

Figure 6. Effect of bubble size and energy dissipation rate per unit mass on the specific breakage rate, $\Omega_B/[(1-\epsilon y_d)n]$, for the oil-water system.

Interfacial tension oil-water $\sigma_{12} = 0.05$ N/m.

This model is geometry independent and predicts breakage rates in the middle of the other models. At droplet sizes above 0.2 mm the predicted rates are higher than those from the model of Laso et al. (1987). For smaller droplet sizes the present predictions are lower than those of Laso et al. (1987), but in this range higher than the model of Coulaloglou and Tavlarides (1977). It has not been possible to run a direct comparison with the models of Chatzi and Kiparissides (1992) because of lack of parameter values. However, extrapolating semiquantitatively their own comparisons from the model of Laso et al. (1987), their model seems to give values below that model above 0.2-mm drop size and fall off to low breakage rate values, very similar to the model presented in this work, at low droplet sizes.

To improve the possibilities for verification of the various models, independent droplet/bubble breakup studies are needed, both concerning the impact of shear and turbulence.

Conclusions

A theoretical fluid-particle (drops and bubbles) breakup-rate model has been developed, based on the theories of probability and turbulence. The breakage-rate model has no unknown parameters since all the constants in the model are determined from isotropic turbulence theory. This is in contrast to previous models that have at least three unknown parameters. In addition, the developed model does not need a predefined daughter particle size distribution, as it directly gives the breakage rate for a given combination of daughter sizes. Also, the daughter particle size distribution can directly be derived from the breakage-rate model.

The breakage fractions predicted by the present model for the air–water system in high-intensity pipeline flow are shown to be in very good agreement with the experimental results of Hesketh et al. (1991a,b). The developed model for specific breakage rate has been compared to models found in the literature, and is found to give breakage-rate values bracketed by the other models. The spread in predictions is, however, large, and improved experimental studies are recommended.

Notation

c_1,c_2 = unknown constants in breakage rate models
c_3,c_4 = constants defined by Eqs. 16 and 18
D = diameter of impeller, m
e = energy of individual eddies, J
$\bar{e}$ = mean of e, J
$\bar{e}_i$ = increase of surface energy due to a bubble breakage, J
n = number of bubbles or drops per unit dispersion volume, m^{-3}
P_e = energy probability
$\hat{u}$ = friction velocity, $(t_W/\rho_c)^{1/2}$, m/s

Greek letters

β = constant defined in Eq. 12
$\Gamma(\ )$ = gamma function
ϵ = energy dissipation rate per unit mass, $m^2 \cdot s^{-3}$
λ_d = eddy size of viscous dissipation, m
μ_c = viscosity of continuous phase, Pa·s
μ_d = viscosity of dispersed phase, Pa·s
ρ_c = density of continuous phase, kg/m^3
ρ_d = density of dispersed phase, kg/m^3
σ = surface tension, N/m

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